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Proofs

[1] Fuzzy eqn's for De Morgain

a) $1-\max[\mu_A(x), \mu_B(x)] = \min[1-\mu_A(x), 1-\mu_B(x)]$

Sol

assume :- $\mu_A(x) \leqslant \mu_B(x)$

$L.H.S = 1-\max[\mu_A(x), \mu_B(x)] = 1-\mu_B(x)$

$\mu_A(x) \leqslant \mu_B(x) \Rightarrow -\mu_A(x) \geqslant -\mu_B(x)$

$1-\mu_A(x) \geqslant 1-\mu_B(x)$

$R.H.S = \min[1-\mu_A(x), 1-\mu_B(x)]$

$= 1-\mu_B(x) = L.H.S$

$\therefore L.H.S = R.H.S$

هو نفسه القانون الآتي:

$(A \cup B)^c = A^c \cap B^c$

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b)

$$1-\min\left[\mu_A(x),\mu_B(x)\right]=\max\left[1-\mu_A(x),1-\mu_B(x)\right]$$

Let: $\mu_A(x) \leq \mu_B(x)$

$$L.H.S = 1-\min\left[\mu_A(x),\mu_B(x)\right]$$

$$L.H.S = 1-\mu_A(x)$$

$$-\mu_A(x) \geq -\mu_B(x) \Rightarrow 1-\mu_A(x) \geq 1-\mu_B(x)$$

$$\therefore \max\left[1-\mu_A(x),1-\mu_B(x)\right] = R.H.S$$

$$R.H.S = 1-\mu_A(x)$$

$$\therefore R.H.S = L.H.S$$

ده نفسه القانون ~~المتقاطع~~

$$(A\cap B)^c = A^c \cup B^c$$

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$\boxed{2}$ show that Yager fuzzy Complement and sugeno satisfies Complement Axioms.

Axioms are :-

$\boxed{1}$ $C(1) = 0, \quad C(0) = 1$

$\boxed{2}$ $a = \mu_A(x) \; ; \; b = \mu_B(x)$

$a \leq b \Rightarrow C(a) \geq C(b)$

A) Sugeno

$$C_\lambda(a) = \frac{1-a}{1+\lambda a} \quad ; \quad a = \mu_A(x)$$

$$C(0) = \frac{1-0}{1+0} = 1 \quad ; \quad C(1) = \frac{1-1}{1+\lambda} = 0 \quad \#$$

$a \leq b \Rightarrow \lambda a \leq \lambda b \quad ; \quad \lambda > 0$

$\therefore 1 + \lambda a \leq 1 + \lambda b$

$$\frac{1}{1+\lambda a} \geq \frac{1}{1+\lambda b} \longrightarrow (1)$$

$a \leq b \Rightarrow -a \geq -b \Rightarrow 1-a \geq 1-b \rightarrow (2)$

Multiply (1) & (2)

$$\frac{1-a}{1+\lambda a} \geq \frac{1-b}{1+\lambda b} \Rightarrow c(a) \geq c(b) \#$$

→ sugeno satisfy complement axioms.

B) Yager

$$C_w(a) = (1-a^w)^{\frac{1}{w}}$$

$$W_w(0) = (1-0^w)^{\frac{1}{W}} = 1 \quad ; \quad C_w(1) = (1-1^w)^{\frac{1}{w}} = 0$$

$$a \leq b \Rightarrow C_w(a) \geq C_w(b)$$

$$a \leq b \rightarrow a^w \leq b^w \Rightarrow -a^w \geq -b^w$$

$$1-a^w \geq 1-b^w \Rightarrow (1-a^w)^{\frac{1}{w}} \geq (1-b^w)^{\frac{1}{w}}$$

$$\therefore C_w(a) \geq C_w(b)$$

⇒ Yager satisfy complement axioms.

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$\boxed{3}$ show that Dombi union operation satisfy s-norm axioms.

Dombi class:

$$S_\lambda(a,b) = \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$a = \mu_A(x)$, $b = \mu_b(x)$

$\boxed{1}$ $s(1,1) = 1$, $s(0,a) = s(a,0) = a$

$s(1,1) = 1$

$$s(0,a) = \frac{1}{1+\left[0+\left(\frac{1}{a}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$$= \frac{1}{1+\left(\frac{1}{a}-1\right)} = \boxed{a}$$

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$$s(a,0) = \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}} = a$$

$$\therefore s(a,0) = s(0,a) = a \quad \# \boxed{1}$$

$\boxed{2}$ $s(a,b) = s(b,a)$

$$s(a,b) = \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$$= \frac{1}{1+\left[\left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{a}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}} = s(b,a) \quad \# \boxed{2}$$

$\boxed{3}$ $a \leq \grave{a}$, $b \leq \grave{b}$

$$\therefore s(a,b) \leq s(\grave{a},\grave{b})$$

$$a \leq \grave{a} \Rightarrow \frac{1}{a} \geq \frac{1}{\grave{a}}$$

$$\frac{1}{a}-1 \geq \frac{1}{\grave{a}}-1 \Rightarrow \left(\frac{1}{a}-1\right)^{\lambda} \geq \left(\frac{1}{\grave{a}}-1\right)^{\lambda}$$

$$\left(\frac{1}{a}-1\right)^{-\lambda} \leqslant \left(\frac{1}{a'}-1\right)^{-\lambda}$$

بالمثل $\left(\frac{1}{b}-1\right)^{-\lambda} \leqslant \left(\frac{1}{b'}-1\right)^{-\lambda}$

$$\therefore \underbrace{\left(\frac{1}{a}-1\right)^{-\lambda} + \left(\frac{1}{b}-1\right)^{-\lambda}}_{*} \leqslant \underbrace{\left(\frac{1}{a'}-1\right)^{-\lambda} + \left(\frac{1}{b'}-1\right)^{-\lambda}}_{**}$$

$$\left[\ *\ \right]^{\frac{-1}{\lambda}} \geqslant \left[**\right]^{\frac{-1}{\lambda}}$$

$$1+\left[*\right]^{\frac{-1}{\lambda}} \geqslant \left[**\right]^{\frac{-1}{\lambda}}$$

$$\frac{1}{1+\left[*\right]^{\frac{-1}{\lambda}}} \cancel{\geqslant} \leqslant \frac{1}{1+\left[**\right]^{\frac{-1}{\lambda}}}$$

$$\Rightarrow S(a,b) \leqslant S(a',b') \ \# \ \boxed{3}$$

$\boxed{4}$ $S\left(S(a,b),c\right) = S\left(a, S(b,c)\right)$?

$$L.H.S = S\left(S(a,b),c\right) = \frac{1}{1+\left[\left(\frac{1}{S(a,b)}-1\right)^{-\lambda} + \left(\frac{1}{c}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$$\frac{1}{S(a,b)} = 1+\left[\left(\frac{1}{a}-1\right)^{-\lambda} + \left(\frac{1}{b}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}$$

$$\left(\frac{1}{S(a,b)}-1\right)^{-\lambda} = \left(\frac{1}{a}-1\right)^{-\lambda} + \left(\frac{1}{b}-1\right)^{-\lambda}$$

$$L.H.S = \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda} + \left(\frac{1}{b}-1\right)^{-\lambda} + \left(\frac{1}{c}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$$R.H.S = S\left(a, S(b,c)\right)$$

$$= \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda} + \left(\frac{1}{S(b,c)}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

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$$S(b,c) = \frac{1}{1 + \left[\left(\frac{1}{b} - 1\right)^{-\lambda} + \left(\frac{1}{c} - 1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$$\frac{1}{S(b,c)} = 1 + \left[\left(\frac{1}{b} - 1\right)^{-\lambda} + \left(\frac{1}{c} - 1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}$$

$$\left(\frac{1}{S(b,c)} - 1\right)^{-\lambda} = \left(\frac{1}{b} - 1\right)^{-\lambda} + \left(\frac{1}{c} - 1\right)^{-\lambda}$$

$$R.H.S = \frac{1}{1 + \left[\left(\frac{1}{a} - 1\right)^{-\lambda} + \left(\frac{1}{b} - 1\right)^{-\lambda} + \left(\frac{1}{c} - 1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$R.H.S = L.H.S$ # $\boxed{4}$

→ Dombi union satisfy S-norm axioms.

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Convex مسائل من السكشن على ا قانون ال

$\boxed{4}$ Convex

$$\mu\left[\lambda x_1 + (1-\lambda) x_2\right] \geq \min\left[\mu_{x_1}, \mu_{x_2}\right]$$

for example: $\mu = \dfrac{1}{1+x^2}$

Let $\mu_{x_1} < \mu_{x_2}$

$$R.H.S = \min(\mu_{x_1}, \mu_{x_2}) = \mu_{x_1} = \frac{1}{1+x_1^2} \longrightarrow (1)$$

$$L.H.S = \mu(\lambda x_1 + (1-\lambda)x_2)$$

$$= \frac{1}{1+\left[\lambda x_1 + (1-\lambda)x_2\right]^2} = \frac{1}{1+(\lambda x_1 + x_2 - \lambda x_2)^2}$$

replace $x_2 \longrightarrow x_1$

$$L.H.S = \frac{1}{1+(\lambda x_1 + x_1 - \lambda x_1)^2} = \frac{1}{1-x_1^2} \longrightarrow (2)$$

$L.H.S = R.H.S \qquad \therefore \mu \longrightarrow$ is Convex

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[2]
$$\mu = \begin{cases} 0 & x \leqslant 10 \\ \dfrac{1}{1+(x-10)^{-2}} & x > 10 \end{cases}$$

$\boxed{Sol}$

at $\mu = 0$ $\quad \mu_{x_1} < \mu_{x_2}$

$R.H.S = \min[\mu_{x_1}, \mu_{x_2}] = \mu_{x_1} = 0$

$L.H.S = \mu[\lambda x_1 + (1-\lambda) x_2] = 0 \longrightarrow$ Convex

for $\mu = \dfrac{1}{1+(x-10)^{-2}}$

$\mu_{x_1} < \mu_{x_2}$

$R.H.S = \min[\mu_{x_1}, \mu_{x_2}] = \mu_{x_1} = \dfrac{1}{1+(x_1-10)^{-2}}$

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L.H.S $= \mu\left[\lambda x_1 + (1-\lambda)x_2\right]$

$$= \frac{1}{1+(\lambda x_1 + (1-\lambda)x_2 - 10)^{-2}} \quad \text{replace } x_2 \to x_1$$

$$= \frac{1}{1+(x_1-10)^{-2}} = R.H.S \quad \therefore \text{Convex}$$